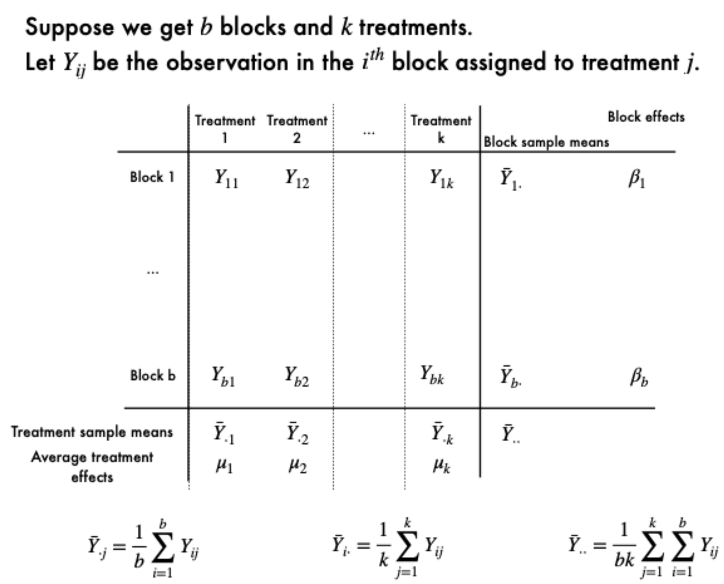
Discussion section 8

Math 181B

1 Review

1.1 Randomized Block Design

From the lecture slides, we have



Suppose we get b blocks and k treatments.
Let Y_{ij} be the observation in the i^{th} block assigned to treatment j.

	Treatment 1	Treatment 2	...	Treatment k	Block sample means	Block effects
Block 1	Y_{11}	Y_{12}		Y_{1k}	$\bar{Y}_{1\cdot}$	β_1
...						
Block b	Y_{b1}	Y_{b2}		Y_{bk}	$\bar{Y}_{b\cdot}$	β_b
Treatment sample means	$\bar{Y}_{\cdot 1}$	$\bar{Y}_{\cdot 2}$		$\bar{Y}_{\cdot k}$	$\bar{Y}_{\cdot\cdot}$	
Average treatment effects	μ_1	μ_2		μ_k		

$$\bar{Y}_{\cdot j} = \frac{1}{b}\sum_{i=1}^{b} Y_{ij} \qquad \bar{Y}_{i\cdot} = \frac{1}{k}\sum_{j=1}^{k} Y_{ij} \qquad \bar{Y}_{\cdot\cdot} = \frac{1}{bk}\sum_{j=1}^{k}\sum_{i=1}^{b} Y_{ij}$$

Figure 1: The illustration of the RBD

We assume

$$Y_{ij} = \mu_j + \beta_i + \varepsilon_{ij}$$

where ε_{ij} are *i.i.d.* normally distributed with mean zero and variance σ^2, for $i = 1, 2, \ldots, b$ and $j = 1, 2, \ldots, k$.

We define

- Total sum of squares: $\text{SSTOT} = \sum_{j=1}^{k}\sum_{i=1}^{b}\left(Y_{ij} - \bar{Y}_{..}\right)^2$.
- Treatment sum of squares: $\text{SSTR} = \sum_{j=1}^{k}\sum_{i=1}^{b}\left(\bar{Y}_{.j} - \bar{Y}_{..}\right)^2 = \sum_{j=1}^{k} b\left(\bar{Y}_{.j} - \bar{Y}_{..}\right)^2$.
- Block sum of squares: $\text{SSB} = \sum_{i=1}^{b}\sum_{j=1}^{k}\left(\bar{Y}_{i.} - \bar{Y}_{..}\right)^2 = \sum_{i=1}^{b} k\left(\bar{Y}_{i.} - \bar{Y}_{..}\right)^2$.

- Error sum of squares: SSE $= \sum_{j=1}^{k} \sum_{i=1}^{b} \left(Y_{ij} - \bar{Y}_{.j} - \bar{Y}_{i.} + \bar{Y}_{..}\right)^2$.

Some facts:

- SSTOT = SSTR + SSB + SSE.
- SSTR, SSB, and SSE are independent random variables.
- When $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$ is true, SSTR $/\sigma^2$ has a chi square distribution with $k-1$ degrees of freedom.
- When $H_0 : \beta_1 = \beta_2 = \ldots = \beta_b$ is true, SSB$/\sigma^2$ has a chi square distribution with $b-1$ degrees of freedom.
- Regardless of whether the μ_j 's and/or the β_i 's are equal, SSE $/\sigma^2$ has a chi square distribution with $(b-1)(k-1)$ degrees of freedom.
- When $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$ is true,

$$F = \frac{\text{SSTR}/(k-1)}{\text{SSE}/(b-1)(k-1)}$$

  has an F distribution with $k-1$ and $(b-1)(k-1)$ degrees of freedom.
- When $H_0 : \beta_1 = \beta_2 = \ldots = \beta_b$ is true,

$$F = \frac{\text{SSB}/(b-1)}{\text{SSE}/(b-1)(k-1)}$$

  has an F distribution with $b-1$ and $(b-1)(k-1)$ degrees of freedom.