Week 2 - Discussion

Problem 1. You will practice applying Bayes rule in this problem.

Suppose we have 3 cards C_1, C_2 and C_3 identical in form. The both sides of C_1 are all red. The both sides of C_2 are all blue. One side of C_3 is blue and the another side is red.

- Randomly select one card and put it down on the ground, use the tree diagram to show the probability that the upper side of this card is Red.
- If you find the upper side of this chosen card is colored red, what is the probability that the other side is colored blue? (Consider applying Bayes rule and Law of total probability)

Solution:

$$\begin{aligned} P(RB|R) &= \frac{P(RB \bigcap R)}{P(R)} \\ &= \frac{P(R|RB)P(RB)}{P(R)} \\ &= \frac{P(R|RB)P(RB)}{P(R|RR)P(RR) + P(R|RB)P(RB) + P(R|BB)P(BB)} \\ &= \frac{\frac{1}{2} \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} \\ &= \frac{1}{3} \end{aligned} \tag{1}$$

Problem 2. This problem will be based on the *A Failure to Communicate* part in the Lecture Slides.

a. Fulfill the below R function to mimic the failure to communicate process in the Lecture Slides. In other words, this function takes 0 or 1 as input, and returns the received value based on the same probability defined in the Lecture. Recall that 0 will be corrupted to a 1 with probability 1/3 and 1 will be corrupted to a 0 with probability 1/5. (You will learn the usage of *if...else...* and *sample(·)* commands)

```
Communicate = function(input = 0){
if(input == 0){
    receive = [Fulfill your code]
}else{
    receive = [Fulfill your code]
}
  return(receive)
}
```

We still assume that the ratio of 0s to 1s sent out is 2:7. As we compute in the Lecture, if a 1 is received by a sever, a 1 is sent by the sender with probability 42/47 ≈ 0.894. Now, we want to test this result with R code.

b. Generate a sent vector with 200000 size using *sample(·)* function.

c. Get the received vector with your function defined in part (a) and the sent vector obtained from part (b). (You may use the *sapply(·)* function or *for* loop)

d. Compute N_1 : the number of sent 1 value with the corresponding received value is 1; N_0 : the number of sent 0 value with the corresponding received value is 1. Compare $\frac{N_1}{N_0+N_1}$ with 0.894.