Week 5 - Discussion

Problem 1. There is a Covid-19 drive-through testing point near the graduate housing. I observe that there are three cars come to do tests on average in a minute. Assume the situation for each minute is independent.

a. We want to analyze how many cars come to do tests in an hour. Find the expected value and standard deviation of this corresponding random variable [WISE].

b. Does this random variable you defined in part (a) have the same behavior with a random variable $Y \sim \text{Poisson}(180)$. Check your answer in R (You might be able to check it by applying hist$(\cdot)$ function).

c. For checking part(b) in a more regular way, you can start from showing the sum of two independent poisson distributions $X \sim \text{Poisson}(\lambda_1)$ and $Y \sim \text{Poisson}(\lambda_2)$ is a poisson distribution with average rate $\lambda_1 + \lambda_2$. Hint: for any $k \geq 0$, we notice that $P(X+Y=k) = \sum_{i=0}^{k} P(Y=k-i, X=i)$.

Problem 2. We have also learned that the sum of independent random variables may have an approximate normal distribution. Thus, we can take a different approach to analyze Problem 1.

a. Guess the approximate normal distribution of the number of cars coming to do tests in an hour. Similarly, check your guess in R.

b. Apply the approximate distribution you found in part (a) to compute the probability that there are more than 220 cars coming to do tests in an hour [Tables and R, assume we know $P(Z < -2.98) = 0.0014$ from table].

c. Compute the probability of the same event in part (b) through Poisson(180) distribution by R. Is this result consistent with part (b).

d. Notice that $P(X > N) = P(X \geq N + 1)$ for a poisson distribution. Can you propose a more accurate way to approximate the probability appearing in part (b) through normal distribution. Actually, you are exploring the continuity correction of poisson distribution.